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**Instructions:** Write complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers

1. Determine whether or not is a conservative vector field. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$

$$\mathbf{F}(x, y) = e^x \sin y \mathbf{i} + e^y \cos x \mathbf{j}$$

2. Find a potential function  $f$  for the vector field  $\mathbf{F}$ , then use it to find the value of

$$\int_C \mathbf{F} \cdot d\mathbf{r} \text{ Where } \mathbf{F} \text{ and } C \text{ are given below}$$

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}, \text{ and}$$

$C$ : the line segment from  $(0, 1, 2)$  to  $(1, 2, 4)$

3. Show that the line integral is independent of path and evaluate the integral.

$$\int_C \sin y dx + (x \cos y - \sin y) dy$$

where  $C$  is the path from  $(2,0)$  to  $(1,\pi)$

4. Let  $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j} = \mathbf{j} \frac{y\mathbf{i} - x\mathbf{j}}{x^2 + y^2}$

a. Show that  $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

b. Show that  $\mathbf{F}$  is not independent of path by computing  $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$  and  $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$  and  $C_1, C_2$  are the upper and lower halves of the circle  $x^2 + y^2 = 1$ .

c. Does the results from a and b contradict the conclusion of theorem 6? Explain.